

# DETERMINATION OF MATERIAL PARAMETERS FOR THE 8-CHAIN MODEL FOR USE WITH ABAQUS, LS-DYNA AND ANSYS

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## Abstract

The 8-chain model [Arruda and Boyce, 1993] has been shown to accurately capture the large strain equilibrium response of many different types of elastomers. This report summarizes a simple graphical technique that can be used to find the necessary material parameters for the 8-chain from uniaxial testing results. Note, that a number of important extensions to the original 8-chain model has been developed. These extensions allow for predictions of different volume fraction fillers [Bergström and Boyce, 1999] and direct predictions of time-dependence and hysteresis [Bergström and Boyce, 1998, 2000].

## 1 Original 8-chain Model

The original 8-chain model can be written as follows:

$$\mathbf{T} = \frac{\mu_0}{J\lambda^{chain}} \cdot \frac{\mathcal{L}^{-1}(\lambda^{chain}/\lambda^{lock})}{\mathcal{L}^{-1}(1/\lambda^{lock})} \text{dev}[\mathbf{B}^*] + \kappa[J - 1]\mathbf{1}, \quad (1)$$

where

- $\mathbf{T}$  is the Cauchy stress (also called the *true stress*)
- $\mu_0$  is the initial shear modulus of the material
- $J = \det[\mathbf{F}]$  is the determinant of the deformation gradient
- $\lambda^{chain}$  is an effective stretch measure that is given by  $\lambda^{chain} = (\text{tr}[\mathbf{B}^*]/3)^{1/2}$ .
- $\mathbf{B}^* = J^{-2/3}\mathbf{F}\mathbf{F}^T$
- $\mathcal{L}^{-1}(x)$  is the inverse Langevin function which can accurately be approximated by

$$\mathcal{L}^{-1}(x) \approx \begin{cases} 1.31446 \tan(1.58986x) + 0.91209x, & \text{if } |x| < 0.84136 \\ 1/(\text{sign}(x) - x), & \text{if } 0.84136 \leq |x| < 1 \end{cases}$$

- $\kappa$  is the bulk modulus of the material.

This tensor equation relates the applied deformation to the true stress acting on the material. For the special case of uniaxial loading the stress-strain response becomes:

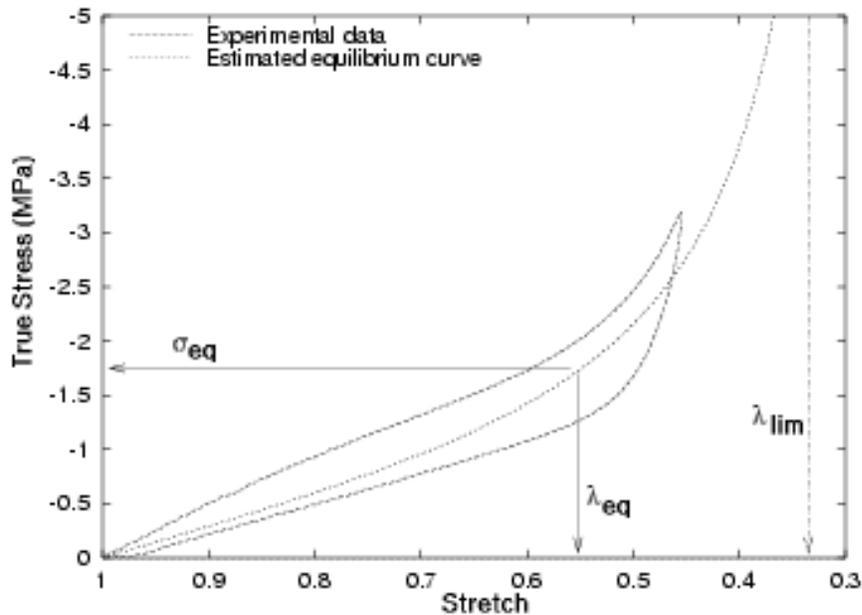
$$\sigma = \frac{\mu_0}{\lambda^{chain}} \cdot \frac{\mathcal{L}^{-1}(\lambda^{chain}/\lambda_{lock})}{\mathcal{L}^{-1}(1/\lambda_{lock})} \left[ \lambda^2 - \frac{1}{\lambda} \right]. \quad (2)$$

The material properties needed by the 8-chain model are:

- the initial shear modulus:  $\mu_0$ ,
- the locking stretch  $\lambda^{lock}$ ,
- the bulk modulus  $\kappa$ .

The following procedure can be used to determine these material parameters for a specific material.

1. Perform a large strain uniaxial tension or compression experiment on the elastomeric material. Plot true stress as a function of applied stretch, see Figure 1. The applied



**Figure 1:** Definition of equilibrium locus and limiting stretch.

stretch is here defined by  $\lambda = L/L_0$ , and the true stress is defined by  $\sigma = \lambda \cdot F/A_0$ .

2. The initial slope of the stress-stretch curve gives the Young's modulus of the material, i.e.  $E = d\sigma/d\lambda$ . Since most elastomers are virtually incompressible, it is possible to estimate the initial shear modulus from  $\mu_0 = E/3$ .
3. The locking stretch  $\lambda^{lock}$  can be obtained from the limiting chain stretch,  $\lambda_{lim}$ , which is the stretch at which the stress starts to increase without limit (see Figure 1), the value of  $\lambda^{lock}$  can be obtained from:

$$\lambda^{lock} = \sqrt{\frac{1}{3} \left[ \lambda_{lim}^2 + \frac{2}{\lambda_{lim}} \right]}.$$

4. The bulk modulus  $\kappa$  cannot be obtained from simple uniaxial testing. In fact, it is difficult to accurately determine the bulk modulus for an elastomer since the material is almost incompressible. In many practical applications it is sufficiently good approximation is to take the bulk to be 100 to 1000 times larger than the shear modulus.

Note that the graphical technique described here is only capable of determining approximate values for the material parameters. To get the best possible fit with the experimental data it is necessary to either use a trial-and-error approach or a numerical optimization technique.

## 2 ABAQUS Implementation of the 8-chain Model

The ABAQUS implementation of the 8-chain model is very similar to the original model representation. The main difference is that ABAQUS uses a fifth-order series expansion of the inverse Langevin function that does not capture the correct behavior as the chain stretch approaches the limiting chain stretch. The difference between the mathematically correct implementation of the inverse Langevin function and the series expansion that is used by ABAQUS, however, is small in most loading conditions.

The material parameters used by ABAQUS are the following:

- initial shear modulus,  $\mu$
- locking stretch,  $\lambda_m$
- volumetric parameter,  $D = 2/\kappa$

All of these parameters can be obtained following the procedure outlined in Section 1.

### 3 LS-DYNA Implementation of the 8-chain Model

The LS-DYNA implementation of the 8-chain model is almost identical to the ABAQUS implementation. The material parameters used by LS-DYNA are the following:

- shear modulus,  $G$
- number of statistical links,  $N = (\lambda^{lock})^2$
- bulk modulus,  $K$

All of these parameters can be obtained following the procedure outlined in Section 1.

### 4 ANSYS Implementation of the 8-chain Model

The ANSYS implementation of the 8-chain model is almost identical to the ABAQUS implementation. The material parameters used by ANSYS are the following:

- shear modulus,  $\mu$
- limiting network stretch,  $\lambda_L$
- incompressibility parameter,  $d = 2/\kappa$

All of these parameters can be obtained following the procedure outlined in Section 1.

## References

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